THE MOTION OF A GAS UNDER THE ACTION OF A PRESSURE ON A PISTON, VARYING ACCORDING TO A POWER LAW

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An integro-differential equation is derived for the plane motion of a gas under the action of a pressure on a piston, varying according to a power law. This same equation describes the motion under the action of a sharp blow. An expansion is found which provides a good approximation for the solution of this problem.

Consideration is given to the behavior of the hydrodynamic quantities close to the piston in a particular example where $\gamma = 7/5$. It is shown that, with the automodelling (similarity) exponent corresponding to a sharp blow, in addition to the known solution of motion under the action of a sharp blow, there exists a solution describing motion under the action of a decreasing pressure on the piston.

In her paper [1], Krasheninnikova considered the similar problem of the motion of a gas under the action of a piston, the speed of which varied according to a power law. In it the plane, cylindrical and spherical cases were studied. We limit ourselves to the plane case. The choice of the pressure on the piston as the determining parameter enables us to describe at once a wide class of motions from the uniformly moving piston to the so-called sharp blow [2-4]. In a series of works [5-9] described in papers by Weizsacker, the problem is considered of discovering a similarity solution of the equations of hydrodynamics in the plane case, to which solutions corresponding to arbitrary initial conditions would tend as time increases indefinitely. The solution obtained in those papers coincides with the solution which is called the "sharp blow" solution in the papers [2-4] and also in the present paper.

1. Statement of the problem. Let us consider the motion of a gas under the action of a pressure p on a piston, decreasing or increasing according to the power law:

$$p = It^{-\alpha} \qquad (I = p_0 t_0^{\alpha}) \tag{1.1}$$

Here t is the time and I a certain constant. Such a motion for an initially cold gas is a similarity (automodelling) motion. It is determined by the four parameters y, t, ρ, I, of which three have independent dimensions (y is the coordinate, ρ_0 the density of the cold gas in front of the piston). Let Y be the coordinate of the front of the shock wave ahead of the piston. Then

$$\frac{dY}{dt} = D = \sqrt{\frac{\gamma+1}{2}\frac{p}{\rho_0}}$$

Hence

$$Y = \sqrt{\frac{\gamma+1}{2}\frac{I'}{\rho_0}}\,\frac{2}{2-\alpha}\,t^{1-{}^1\!/_2\alpha} \tag{1.2}$$

Here we have first made use of the fact that it is a similarity motion, since we have assumed that the pressure at the front of the shock wave depends upon time in the same way as the pressure on the piston, differing from it only in the values of the constants I and I'.

Let us raise equation (1.2) to the power $n = 2\alpha/(2-\alpha)$ and, making use of (1.1), we obtain

$$p = A\rho_0 Y^{-n}, \qquad A = \left(\sqrt{\frac{\gamma+1}{2}\frac{I'}{\rho_0}}\,\frac{2}{2-\alpha}\right)^{-n}(I\rho_0)^{-1}$$

Let y be the Lagrangian coordinate. Introducing the independent variable $x = y/Y$, varying between the limits 0 and 1, we can represent the pressure, velocity and density in the form

$$p = A\rho_0 Y^{-n} f(x), \quad u = \sqrt{A}\, Y^{-{}^1\!/_2 n} v(x), \qquad \rho = \rho_0 q(x) \tag{1.3}$$

At the front of the shock wave, $x = 1$. Let us choose the function $f(x)$ so that $f(1) = 1$. Then $v(1) = \sqrt{2/(\gamma+1)}$. Using the equations of gas dynamics in Lagrange's form, we can obtain a system of equations for f, v [3]

$$\frac{n}{2}v + x\frac{dv}{dx} = \sqrt{\frac{2}{\gamma+1}}\frac{df}{dx}$$
$$\frac{dv}{dx} = -\sqrt{\frac{\gamma+1}{2}}\,\frac{\gamma-1}{\gamma+1}\,x\,\frac{d}{dx}[x^n f]^{-\frac{1}{\gamma}} \tag{1.4}$$

and an explicit relationship for the reduced density q:

$$q = \frac{\gamma+1}{\gamma-1}(x^n f)^{\frac{1}{\gamma}} \tag{1.5}$$

Integrating the second equation (1.4) under the condition that

$v(1) = \sqrt{2/(\gamma + 1)}$, we have

$$v = \sqrt{\frac{2}{\gamma+1}}\left\{\frac{\gamma+1}{2} - \frac{\gamma-1}{2}\int_x^1 (x^n f)^{-\frac{1}{\gamma}}\,dx - \frac{\gamma-1}{2}\,x\,(x^n f)^{-\frac{1}{\gamma}}\right\} \qquad (1.6)$$

Substituting v in the first equation (1.4), we obtain the fundamental equation of the problem under consideration

$$\frac{n}{4}(\gamma+1) - \frac{n}{4}(\gamma-1)\int_x^1 (x^n f)^{-\frac{1}{\gamma}}\,dx + \frac{n}{4}(\gamma-1)\left(\frac{2}{\gamma}-1\right)x\,(x^n f)^{-\frac{1}{\gamma}} +$$

$$+\left[\frac{\gamma-1}{2\gamma}(x^n f)^{-\frac{1}{\gamma}-1} x^{n+2} - 1\right]\frac{df}{dx} = 0 \qquad (1.7)$$

with the boundary condition $f = 1$ when $x = 1$.

2. Solutions for certain individual cases. Let us try to solve the problem for all values of γ and n. Let us investigate the general behavior of the function $f(x)$ for different values of n. If the pressure on the piston falls, then the pressure in the gas will decrease from the shock wave to the piston the more sharply, the faster the rate of pressure drop on the piston. If $n = 0$, i.e. for constant pressure, $f =$ const$=$ 1. Accordingly, the set of solutions for an arbitrary value of γ and all values of n can be represented in the form of a pencil of curves radiating from the point $f = 1$, $x = 1$ (Fig. 1).

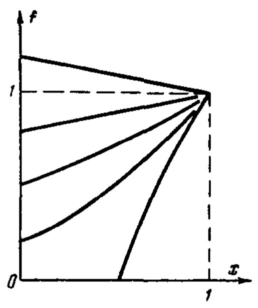

Fig. 1.

The horizontal line corresponds to $n = 0$, i.e. constant pressure. The curves lying above correspond to negative values of n, i.e. the pressure on the piston increases with time.

As n increases, the ratio of the pressure at the shock to the pressure on the piston increases. For a certain value of n the pressure on the piston (more precisely, at the point $x = 0$) becomes equal to zero and the ratio $p(1)/p(0)$ tends to infinity.

Such a pressure distribution corresponds to a sharp blow, as defined in Zel'dovich's paper [2].

We shall now seek values of γ and n for which analytic solutions exist. Let us consider the limiting case $\gamma = 1$. Such a substance will be called "isothermal". For this the equation takes the form:

$$\frac{df}{dx} = \frac{n}{2} \tag{2.1}$$

The solution satisfying the boundary condition $f(1) = 1$ is

$$f = 1 + \frac{1}{2} n(x-1) \tag{2.2}$$

Let us turn to the other limiting case of an incompressible liquid, i.e. $\gamma = \infty$. For this it is convenient to pass from γ to the limiting compression h, where $\gamma = (h+1)/(h-1)$. Then equation (1.7) takes the form

$$\frac{n}{2} h - \frac{n}{2} \int_x^1 (x^n f)^{-\frac{h-1}{h+1}} dx + \frac{n}{2} \frac{h-3}{h-1} x (x^n f)^{-\frac{h-1}{h+1}} +$$
$$+ (h-1) \left[\frac{1}{h+1} (x^n f)^{-\frac{2h}{h+1}} x^{n+2} - 1 \right] \frac{df}{dx} = 0$$

Carrying out the necessary transformations and making $h \to 1$, we obtain the equation for incompressible fluid

$$\frac{n}{2}(x+1) + \frac{n^2}{4}(x-1) + \frac{n}{4}\left[\int_x^1 \ln f \, dx + x \ln f\right] - \left(1 - \frac{x^2}{2f}\right)\frac{df}{dx} = 0 \tag{2.3}$$

At the point $x = 1$ the value of the derivative $df/dx = 2n$. Consequently, at points close to $x = 1$, we have

$$f = 1 + 2n(x-1) + \ldots \tag{2.4}$$

Comparing the expression so obtained with formula (2.2), we see that, for a given value of n, the curves describing the variation of pressure in gases with different adiabatic indices γ leave the point $x = 1$ in the form of a pencil (Fig. 2), bounded in the neighborhood of $x = 1$ by the

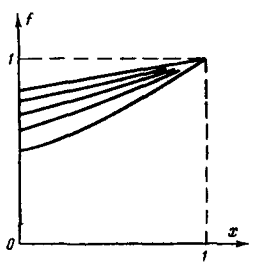

Fig. 2.

straight lines with slopes $1/2n$ and $2n$, corresponding to the two limiting cases $\gamma = 1$ and $\gamma = \infty$.

Let us put $f = x^{\alpha}$, which corresponds to the case of a sharp blow, since $f = 0$ when $x = 0$.

Substituting $f = x^{\alpha}$ in equation (1.7), we have

$$\frac{n}{4}\left[\gamma + 1 - \frac{(\gamma-1)\gamma}{\gamma - n - \alpha}\right] + \\ + (\gamma - 1)\left[\frac{n}{4}\left(\frac{\gamma}{\gamma - n - \alpha} + \frac{2}{\gamma} - 1\right) + \frac{\alpha}{2\gamma}\right] x^{1-\frac{n+\alpha}{\gamma}} - \alpha x^{\alpha-1} = 0$$

In order to satisfy this equation, it is necessary to assume that

$$1 - \frac{n+\alpha}{\gamma} = \alpha - 1, \qquad \gamma + 1 - \frac{(\gamma-1)\gamma}{\gamma - n - \alpha} = 0 \\ (\gamma - 1)\left[\frac{n}{4}\left(\frac{\gamma}{\gamma - n - \alpha} + \frac{2}{\gamma} - 1\right) + \frac{\alpha}{2\gamma}\right] = \alpha \tag{2.5}$$

or

$$\alpha = 1, \qquad \frac{n}{4}\left[\gamma + 1 - \frac{(\gamma-1)\gamma}{\gamma - n - \alpha}\right] = \alpha \\ (\gamma - 1)\left[\frac{n}{4}\left(\frac{\gamma}{\gamma - n - \alpha} + \frac{2}{\gamma} - 1\right) + \frac{\alpha}{2\gamma}\right] = 0 \tag{2.6}$$

The system of equations (2.5) is self-contradictory and does not give any new solution. The system (2.6) at once gives the already known result:

$$\gamma = 1, \qquad n = 2$$

Putting $\gamma \neq 1$, we obtain for γ the equation $5\gamma^2 - 12\gamma + 7 = 0$, which has for its roots 1 and 7/5.

Using the expressions (1.5) and (1.6), we obtain formulas for the complete set of hydrodynamic quantities:

$$f = x, \qquad q = 6x^{5/6}, \qquad v = \frac{1}{2}\sqrt{\frac{5}{6}}\left(3 - \frac{1}{x^{5/6}}\right) \tag{2.7}$$

This solution was obtained by Hoerner [7], Hafele [9] and also by Zhukov and Kazhdan [4].

Accordingly, equation (1.7), describing the motion of a gas under the action of a pressure on a piston, varying according to a power law, has solutions represented by pencils of curves starting from the point $x = 1$, $f = 1$. Moreover, to each value of the parameter n, characterizing the rate of change of the pressure on the piston, there corresponds its own pencil containing the solutions relating to all possible values of the adiabatic index from $\gamma = 1$ to $\gamma = \infty$. The smaller the parameter n in absolute magnitude, the narrower is the pencil. When $n = 0$ the pencil degenerates into a straight line.

Examples of the form and distribution of such pencils in the x, f plane are illustrated in Fig. 3.

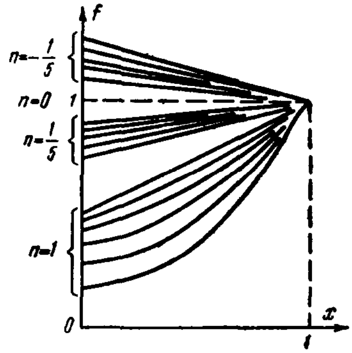

Fig. 3.

All the curves relating to a definite value of n lie in the confines of the angle formed by the lines

$$f = 1 + \tfrac{1}{2} n (x - 1), \qquad f = 1 + 2n (x - 1)$$

Moreover, amongst this set of solutions there is a solution $f = 1 + (x - 1)$, which corresponds to $\gamma = 7/5$ and $n = 4/3$.

All these considerations lead us to expect that a good description of the solution of equation (1.7) in a wide range of values of γ and n can be achieved by the first few terms in the expansion of the function $f(x)$ as a Taylor series near the point $x = 1$.

Let us write down the first four terms of this expansion:

$$\begin{aligned} f = 1 &+ n\frac{2\gamma - 1}{\gamma + 1}(x - 1) + \frac{n}{2}\frac{\gamma - 1}{(\gamma + 1)^2}\left\{5\gamma - 1 - \frac{n}{2}(5\gamma + 2)\right\}(x - 1)^2 + \\ &+ \frac{n}{6}\frac{\gamma - 1}{(\gamma + 1)^3}\left\{2(12\gamma^2 - 11\gamma + 1) - \frac{3}{2}n(35\gamma^2 - 19\gamma - 2) + \right. \\ &\left. + \frac{n^2}{4}(111\gamma^2 - 59\gamma + 4)\right\}(x - 1)^3 + \frac{n}{24}\frac{\gamma - 1}{(\gamma + 1)^4}\left\{6(29\gamma^3 - 47\gamma^2 + 19\gamma - 1) - \right. \\ &- n(784\gamma^3 - 1043\gamma^2 + 272\gamma + 11) + \frac{n^2}{4}(4159\gamma^3 - 4876\gamma^2 - 1165\gamma - 24) - \\ &\left. - \frac{n^3}{8}(3419\gamma^3 - 3762\gamma^2 + 855\gamma + 8)\right\}(x - 1)^4 + \ldots \end{aligned} \tag{2.8}$$

This expansion solves the problem with a sufficient degree of accuracy for practical purposes for all γ and n in the case of decreasing pressure, and for small γ and n in the case of increasing pressure.

From a comparison of the results of exact numerical computation of equations (1.4) with the expansion formula (2.8), it follows that the smaller the value of n and the closer γ lies to unity, the more exact is the expansion. If $n < 2/3$ and $\gamma < 2$ we can limit ourselves to three terms, whilst for even smaller γ and n we can take just the first two terms of the expansion. This expansion can, moreover, be used for finding the greatest possible value of n for any given γ, i.e. the value of n corresponding to the sharp blow. In fact, taking the expressions for f, containing one and then two, three and four terms of the expansion, and putting them equal to zero when $x = 0$, we obtain equations for the first, second, third and fourth approximations, respectively, to the parameter n for the sharp blow. The value of n varies between the limits $n = 1.117$ for $\gamma = \infty$ (incompressible fluid) to $n = 2$ for $\gamma = 1$ ("isothermal" substance). In his paper Hafele presented values of the quantity $k = n/(2+n)$ for four values of γ obtained by numerical methods: $\gamma = 1.1$, $k = 0.43112$; $\gamma = 1.4$, $k = 0.4$; $\gamma = 5/3$, $k = 0.38927$; $\gamma = 2.8$, $k = 0.343296$.

From the expansion (2.8) for f we can obtain the density, using the explicit relation (1.5), and the velocity by means of the quadrature (1.6). We notice that from formula (1.5) it follows immediately that the density at the piston is equal to zero for positive n and infinite for negative n.

3. The construction of similarity solutions. The particular case $n = 1$, $\alpha = 2/3$ coincides with the known solution of Sedov [10] for a strong explosion in plane variant. In fact, in this solution the pressure at the shock and in the whole region decreases as $t^{-2/3}$. In Sedov's solution the motion is considered of a gas on both sides of a plane, at which there is an instantaneous introduction of energy. By virtue of symmetry, the plane at which the energy is introduced remains motionless.

This solution can be interpreted as describing the motion of gas under the action of a pressure on a piston decreasing as $t^{-2/3}$. The piston for such a motion remains motionless. It is evident that such a motion is the borderline between motions of two classes: the first with $\alpha < 2/3$ and $n < 1$, for which the piston moves in the direction of the general motion of the gas (the velocity of motion of the gas in this case does not anywhere vanish); and the second with $\alpha > 2/3$ and $n > 1$, including the case of the sharp blow, for which the piston is withdrawn (in this case the velocity of the gas vanishes at a certain point, the gas moving in both directions).

In order that the motion be similar, the pressure must vary with time according to the law $p = It^{-\alpha}$.

For negative values of α, according to this law the pressure is equal to zero at the instant $t = 0$ and thereafter increases proportionally to

$t^{|a|}$. Such behavior of the pressure as a function of time can be realized in practice, and therefore the motion with the pressure on the piston increasing according to the power law is a similarity flow from the moment of its inception. For positive values of a the pressure at the instant $t = 0$ is infinite and thereafter falls off approximately as t^{-a}.

The infinite pressure at the instant $t = 0$ cannot be realized. In a practical problem we have to assume, for example, that a pressure with amplitude p_0 acts until the time t_0, after which it falls according to the law $p = It^{-a}$.

Let us determine the energy imparted to the gas by the piston during the duration of the motion. In the plane case $E \sim px$, where x is the length of the region covered by the motion, $p \sim t^{-a}$, $x \sim ut \sim t^{1-a/2}$ and consequently $E \sim t^{1-3a/2}$.

The energy delivered by the piston in the duration of the flow with constant pressure is $E_0 \sim t_0^{1-3a/2}$, if we assume that $p_0 \sim t_0^{-1}$.

Hence it follows that when $a < 2/3$ the energy delivered by the piston in the process of the motion according to the law t^{-a} increases indefinitely with time. Accordingly, the fraction of the energy of the gas acquired by the gas during the time of the constant pressure on the piston tends to zero. From this it follows that the motion becomes a similarity flow when $E \gg E_0$ or $t \gg t_0$. In the case $a = 2/3$, the piston, on which the pressure is falling as $t^{-2/3}$, does not impart energy to the gas - it just imparts an impulse. The energy in the gas is acquired during the time of the first shock.

This case is described by Sedov's solution. Sedov also showed that, by the time $t \gg t_0$, the motion becomes a similarity flow. Finally, if a is greater than 2/3 and less than the value of a in the case of the sharp blow, then energy is imparted to the gas in the first shock and thereafter is removed by the piston in the process of its backward motion. Accordingly, in this case the motion for the whole of its duration is essentially determined by two characteristic parameters: the energy imparted by the first shock and the quantity I in the formula $p = It^{-a}$, determining the pressure on the piston. Therefore, without special study, it is impossible to say whether a similarity flow is set up in this case. The establishment of a similarity flow in the case of the sharp blow was demonstrated by Zel'dovich [2], Zhukov and Kazhdan [4].

4. The behavior of the hydrodynamic quantities close to the piston. Let us consider in more detail the behavior of the hydrodynamic quantities near the piston in the case of a motion with $n > 1$ or $a > 2/3$.

For this we shall make use of certain results from paper [3]. In the latter paper equations (1.4) were reduced for the study of the sharp blow to the single equation

$$\frac{du}{dz}=\frac{1}{z}\,\frac{\frac{\gamma-1}{\sqrt{2(\gamma+1)}}n-\frac{\gamma-1}{4}(n+2)u-\frac{n\gamma}{2}\sqrt{\frac{\gamma+1}{2}}u^2z+\gamma uz}{\frac{n}{2}(\gamma+1)\sqrt{\frac{\gamma+1}{2}}uz-(2\gamma-n)z+\frac{\gamma-1}{2}(n+2)} \qquad (4.1)$$

by means of the substitutions

$$f=z^{\frac{\gamma}{\gamma+1}}x^{\frac{2\gamma-n}{\gamma+1}}, \qquad v=uz^{\frac{\gamma}{\gamma+1}}x^{\frac{\gamma-n-1}{\gamma+1}}$$

Here the integral curve corresponding to the sharp blow was found from the condition that it passes through the singular point of the equation. The singular point in the variables u, z has the coordinates

$$u=-2\sqrt{2/\gamma+1}, \qquad z=(\gamma-1)/2\gamma$$

Let us pass now to the independent variable x and the function f. The relation between these quantities at the singular point is

$$f=\left(\frac{\gamma-1}{2\gamma}\right)^{\frac{\gamma}{\gamma+1}}x^{\frac{2\gamma-n}{\gamma+1}} \qquad (4.2)$$

If we know the behavior of the pressure $f(x)$ in the case of the sharp blow, then we can find x corresponding to the singular point. This singular point is a saddle. One of the separatrices of the saddle is the integral curve corresponding to the sharp blow. In this case all the hydrodynamic quantities at the singular point are continuous and smooth.

On the other hand, we can consider the singular point as the site of a weak discontinuity and continue the function $f(x)$ beyond the singular point along the other separatrix. It is then found that $f(x)$ increases with decreasing x and is equal to a finite quantity when $x = 0$. This branch is the solution of the problem of motion under the action of a pressure on the piston decreasing according to the law $p = It^{-\alpha}$, where $\alpha = 2n/(2+n)$ and n is the index of the similarity solution for the sharp blow. The fact that the solution with the other separatrix corresponds to the piston is confirmed by the following argument. In the case $n = 1$ the piston moves backwards (the velocity is negative), slowing down. This means that $\partial u/\partial t > 0$. Then from the equation of motion $\partial u/\partial t = -\rho_0^{-1}\,\partial p/\partial x$, it follows that $\partial p/\partial x < 0$. Accordingly, the general shape of the pressure profile from the shock front to the piston is not monotonic. The pressure profile from the shock front to the piston first of all decreases, then passes through a minimum, and increases again close to the piston. In fact, if we choose the solution for the motion with the piston, a figure is obtained in the form of a broken curve, consisting of two separatrices. As an example, we can cite the case $\gamma = 7/5$, which is convenient as there is an analytical solution for this case.

The singular point in the u, z variables has coordinates $u = -2\sqrt{5/6}$, $z = 1/7$, $f = (1/7)^{7/12} x^{11/18}$, and since in the given case $f = x$, then $x = (1/7)^{3/2} \approx 0.054$.

The branch corresponding to the piston has no analytical solution.

From numerical computations it follows that $f(x)$ increases from $f = 0.054$ at the singular point to $f = 0.1101$ at the piston. Let us notice that $\partial p/\partial x = 0$ at the piston in the case $n = 1$ (Sedov's case) and $\partial p/\partial x > 0$ when $n < 1$.

Accordingly, the pressure profile fails to be monotonic only when $1 < n < n_1$, where n_1 corresponds to the sharp blow.

Let us explain the physical significance of the singular point. For this, we must make a transformation from Lagrangian coordinates to Eulerian. Let ξ be the ratio of the running Eulerian coordinate to the coordinate of the shock front; the transformation formula is

$$\xi = 1 - \int_x^1 \frac{dx}{q} \tag{4.3}$$

For the case $\gamma = 7/5$ and $n = 4/3$, substituting $q = 6x^{5/3}$, we obtain

$$\xi = \frac{5}{4} - \frac{1}{4} x^{-2/3}, \qquad x = (5 - 4\xi)^{-3/2} \tag{4.4}$$

For the hydrodynamic quantities and the adiabatic velocity of sound we obtain

$$f = \frac{1}{(5-4\xi)^{3/2}}, \qquad q = \frac{6}{(5-4\xi)^{5/2}}$$

$$v = \sqrt{\frac{5}{6}}(2\xi - 1), \qquad c = \sqrt{\frac{7}{30}(5 - 4\xi)} \tag{4.5}$$

Let z and Z be the running Eulerian coordinate and that of the shock front, so that $\xi = z/Z$. It is known that $Z = At^{1-\alpha/2}$,

$$Z = At^{3/5}, \qquad \xi = \frac{z}{Z} = \frac{z}{At^{3/5}}$$

The constant A is not difficult to find, using the relations at the front of the shock wave,

$$\frac{dZ}{dt} = \frac{3}{5} At^{-2/5}$$

On the other hand,

$$\frac{dZ}{dt} = D = \sqrt{\frac{6}{5}}\, t^{-2/5}$$

Hence $A = \sqrt{10/3}$ and

$$z = \sqrt{\frac{10}{3}}\, \xi t^{3/5} \tag{4.6}$$

Now let us construct the z - t diagram, on which we shall indicate certain ξ-curves and the field of characteristics. Let us write down the equation of that family of characteristics which carries disturbances to the shock front:

$$\frac{dz}{dt} = u + c$$

We have

$$\frac{dz}{dt} = [u(\xi) + c(\xi)]\, t^{-\frac{\alpha}{2}} \tag{4.7}$$

In the case under consideration, equation (4.7) takes the form

$$\frac{dz}{dt} = \left[\sqrt{\tfrac{5}{6}}(2\xi - 1) + \sqrt{\tfrac{7}{30}(5 - 4\xi)}\,\right] t^{-2/5} \tag{4.8}$$

Giving ξ various actual values, we shall obtain the equations of the ξ-curves and the slopes of the characteristics at their intersection with the corresponding ξ-curves. Putting $\xi = 1$ in formula (4.6), we obtain the equation of the shock front:

$$Z = \sqrt{\tfrac{10}{3}}\, t^{3/5}$$

and the slope with which the characteristics intersect the shock front:

$$\frac{dz}{dt} = \left(\sqrt{\tfrac{5}{6}} + \sqrt{\tfrac{7}{30}}\right) t^{-2/5}$$

The equation of the point at which the motion is generated is also represented by the ξ-curve itself, with $\xi = 0$. The characteristics cut it with slope:

$$\frac{dz}{dt} = \left[-\sqrt{\tfrac{5}{6}} + \sqrt{\tfrac{7}{6}}\right] t^{-2/5}$$

At the singular point $\xi = -1/2$. For the corresponding ξ-curve,

$$z = -\sqrt{\tfrac{5}{6}}\, t^{3/5}, \qquad \frac{dz}{dt} = -\sqrt{\tfrac{3}{10}}\, t^{-2/5}$$

Accordingly, the slope of the characteristic coincides with the slope of the ξ-curve itself, i.e. the ξ-curve corresponding to the singular point is a characteristic.

In Fig. 4 is represented the form of the field of characteristics for the case $\gamma = 7/5$ and $n = 4/3$ (the curve *1* is the front of the shock wave, curve *2* is the ξ-curve on which $dz/dt = 0$, curve *3* is the characteristic coinciding with the ξ-curve, and curve *4* is the cavitation front).

It follows from this field of characteristics that all disturbances, other than shock waves, originating to the left of the curve $\xi = -1/2$, do not penetrate the region to the right of this curve and do not reach the shock front. Accordingly, the region as far as the curve $\xi = -1/2$

on the left can be added not only to the solution corresponding to motion with a piston, but also to the solution corresponding to the sharp blow.

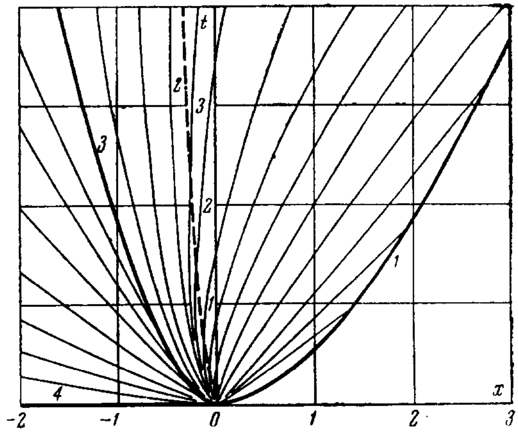

Fig. 4.

We notice that the branching point slides along the curve $f(x)$ as γ varies. For incompressible fluid ($\gamma = \infty$) it occurs very close to the front ($x \sim 0.3$), whilst for an "isothermal" substance ($\gamma = 1$) it disappears.

The pressure at the front of the shock wave cannot fall off more rapidly than for the sharp blow, even if the pressure at the piston undergoes a more rapid fall. This follows from the fact that the curves describing the solution when $n > n_1$ of the sharp blow, on leaving the point (1.1), are located below the curve for the sharp blow and, failing to reach the piston ($x = 0$), turn in the direction of increasing x, as depicted in Fig. 5.

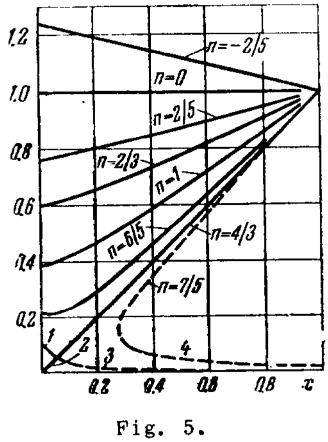

Fig. 5.

Here curve *1* is the branch corresponding to the piston, curve 2 is the branch corresponding to the sharp blow, curves *3* and *4* are branches without physical significance.

The dashed curve is the continuation in the direction of increasing x of the solution describing the motion with a piston when $n = n_1$. A certain physical significance can be ascribed to the branches of these solutions from the point (1.1) up to the turning point. They correspond to the motion which is generated by the action of a sharp blow and such that the gas just behind the shock wave is accompanied by a reservoir capable of absorbing the gas. Moreover, the velocity of this motion is such as to conserve the fraction of gas absorbed by this reservoir, relative to the quantity of gas engulfed by the shock wave, and this is numerically equal to the coordinate x of the turning point.

Different velocities of the motion and different fractions of the gas correspond to solutions with different values of n. Such motions are realized, for example, in the case of a semi-infinite pipe. If a sharp blow impinges on the gas in the pipe from the open end, then a shock wave is propagated along the gas in the pipe, whilst material flows out of the open end of the pipe. Here the space outside the pipe constitutes the reservoir in which the gas is absorbed. In this case the turning point has the Eulerian coordinate $\xi = 0$, i.e. the reservoir is stationary at the point where the motion is generated.

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